Back Paper Exam
B.Math Algebra-IV
2015-2016

Time: 3 hrs
Max score: 100

Answer all questions.

(1) State true or false. Give reasons.
(a) Let $F \subset K \subset L$ be field extensions. If $L|F$ is Galois then so is $K|F$.
(b) Galois group of $x^3 - 2$ over $\mathbb{F}_7$ is cyclic of order 3.
(c) 40 degrees is a constructible angle.
(d) An algebraically closed field must be infinite.
(e) Let F be a field, char(F) not dividing n, containing the nth roots of unity, then the extension $F(\sqrt[n]{a})$ for all $a \in F$ is cyclic. (5× 5)

(2) Determine the splitting field of the polynomial $x^p - x - a$ over $\mathbb{F}_p$ where $a \neq 0$. Show explicitly that the Galois group is cyclic. (10)

(3) Show that the polynomial $x^{p^n} - x$ is precisely the product of all the distinct irreducible polynomials in $\mathbb{F}_p[x]$ of degree d where d runs through all divisors of n. (10)

(4) (i) State the fundamental theorem of Galois theory.
(ii) Compute the Galois group of the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ over $\mathbb{Q}$. (5+7)

(5) Let F be a finite, normal extension of $\mathbb{Q}$ for which $|Gal(F/\mathbb{Q})| = 8$ and each element of $Gal(F/\mathbb{Q})$ has order 2. Find the number of subfields of F that have degree 4 over $\mathbb{Q}$. (10)

(6) (i) Let $K|F$ be a finite extension. Then show that K is a simple extension of F if and only if there exist only finitely many subfields of K containing F.
(ii) Hence show that if $K|F$ is finite and separable, then $K|F$ is simple. (10+5)

Please turn over

(7) (i) Show that the polynomial $p(x) = x^5 - 4x + 2$ is irreducible over $\mathbb{Q}$, and find the number of real roots.
(ii) Find the Galois group of $p(x)$ over $\mathbb{Q}$, and explain why the group is not solvable. (8+10)

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